Corrigé de l'examen de math 1ECCG, du 3.11.2010

1. $(3x-4y)(6x-5y)-[(3x-y)(2x+3)-(5x-3y)(x-2y)]$
$=18x^2-15xy-24xy+20y^2-[6x^2+9x-2xy-3y-(5x^2-10xy-3xy+6y^2)]$
$=18x^2-39xy+20y^2-[6x^2+9x-2xy-3y-5x^2+10xy+3xy-6y^2]$
$=18x^2-39xy+20y^2-[x^2+9x+11xy-3y-6y^2]$
$=18x^2-39xy+20y^2-x^2-9x-11xy+3y+6y^2$
$=17x^2-50xy-9x+26y^2+3y$

2. $(4x^4y^3-7x^5y)^2$
$=(4x^4y^3)^2-2\cdot 4x^4y^3\cdot 7x^5y+(7x^5y)^2$
$=16x^8y^6-56x^9y^4+49x^{10}y^2$

3. $(4x^{m-3}y^3-9x^{3m+2}y^{4m-1})^3$
$=(4x^{m-3}y^3)^3-3\cdot(4x^{m-3}y^3)^2\cdot 9x^{3m+2}y^{4m-1}+3\cdot 4x^{m-3}y^3\cdot(9x^{3m+2}y^{4m-1})^2-(9x^{3m+2}y^{4m-1})^3$
$=64x^{3m-9}y^9-3\cdot 16x^{2m-6}y^6\cdot 9x^{3m+2}y^{4m-1}+3\cdot 4x^{m-3}y^3\cdot 81x^{6m+4}y^{8m-2}-729x^{9m+6}y^{12m-3}$
$=64x^{3m-9}y^9-432x^{5m-4}y^{4m+5}+972x^{7m+1}y^{8m+1}-729x^{9m+6}y^{12m-3}$

4. $(21x^{2m-2}+35x^{4m+1}-14x^{2m-3})\div(7x^{2m-3})$
$=\dfrac{21x^{2m-2}}{7x^{2m-3}}+\dfrac{35x^{4m+1}}{7x^{2m-3}}-\dfrac{14x^{2m-3}}{7x^{2m-3}}$
$=3x^{(2m-2)-(2m-3)}+5x^{(4m+1)-(2m-3)}-2x^{(2m-3)-(2m-3)}$
$=3x^{2m-2-2m+3}+5x^{4m+1-2m+3}-2x^{2m-3-2m+3}$
$=3x^1+5x^{2m+4}-2x^0$
$=5x^{2m+4}+3x-2$

5. $(15x^3y^2+7xy^4)^3$
$=(15x^3y^2)^3+3\cdot(15x^3y^2)^2\cdot 7xy^4+3\cdot 15x^3y^2\cdot(7xy^4)^2+(7xy^4)^3$
$=3'375x^9y^6+3\cdot 225x^6y^4\cdot 7xy^4+3\cdot 15x^3y^2\cdot 49x^2y^8+343x^3y^{12}$
$=3'375x^9y^6+4'725x^7y^8+2'205x^5y^{10}+343x^3y^{12}$

6. $(14x^4-27x^3y+21x^2y^2-3xy^3-2y^4)\div(2x^2-3xy+2y^2)$

$Q=7x^2-3xy-y^2\ ;\ R=0\ ou$
$14x^4-27x^3y+21x^2y^2-3xy^3-2y^4=(7x^2-3xy-y^2)(2x^2-3xy+2y^2)$

$$\begin{array}{rrrrrrrrrrrl}
14x^4 & - & 27x^3y & + & 21x^2y^2 & - & 3xy^3 & - & 2y^4 & & & |\underline{2x^2-3xy+2y^2} \\
-(14x^4 & - & 21x^3y & + & 14x^2y^2 & & & & & &) & 7x^2-3xy-y^2 \\
\hline
 & - & 6x^3y & + & 7x^2y^2 & - & 3xy^3 & - & 2y^4 & & & \\
-(& - & 6x^3y & + & 9x^2y^2 & - & 6xy^3 & & & &) & \\
\hline
 & & & - & 2x^2y^2 & + & 3xy^3 & - & 2y^4 & & & \\
 & & -(& - & 2x^2y^2 & + & 3xy^3 & - & 2y^4 & &) & \\
\hline
 & & & & & & & & 0 & & &
\end{array}$$

7. $(x^6-1)\div(x^3+2x^2+2x+1)$

$Q=x^3-2x^2+2x-1;R=0\ ou$
$x^6-1=(x^3-2x^2+2x-1)(x^3+2x^2+2x+1)$

$$\begin{array}{rrrrrrrrrrrrrl}
x^6 & + & 0x^5 & + & 0x^4 & + & 0x^3 & + & 0x^2 & + & 0x & - & 1 & |\underline{x^3+2x^2+2x+1} \\
-(x^6 & + & 2x^5 & + & 2x^4 & + & x^3 & & & & & &) & x^3-2x^2+2x-1 \\
\hline
 & - & 2x^5 & - & 2x^4 & - & x^3 & + & 0x^2 & + & 0x & - & 1 & \\
-(& - & 2x^5 & - & 4x^4 & - & 4x^3 & - & 2x^2 & & & &) & \\
\hline
 & & & & 2x^4 & + & 3x^3 & + & 2x^2 & + & 0x & - & 1 & \\
 & & & -(& 2x^4 & + & 4x^3 & + & 4x^2 & + & 2x & &) & \\
\hline
 & & & & & - & x^3 & - & 2x^2 & - & 2x & - & 1 & \\
 & & & & -(& - & x^3 & - & 2x^2 & - & 2x & - & 1 &) \\
\hline
 & & & & & & & & & & & & 0 &
\end{array}$$

8. $5x^2-\{(3x-4y)(4x-y)-[4x(3x-2y)-2y(3x+4y)]\}$
$=5x^2-\{12x^2-3xy-16xy+4y^2-[12x^2-8xy-6xy-8y^2]\}$
$=5x^2-\{12x^2-19xy+4y^2-[12x^2-14xy-8y^2]\}$
$=5x^2-\{12x^2-19xy+4y^2-12x^2+14xy+8y^2\}$
$=5x^2-\{-5xy+12y^2\}$
$=5x^2+5xy-12y^2$